Figure 7-17. Conducting sheet carrying a current density of λ amperes per meter. Since $\nabla \cdot \boldsymbol{B} = 0$, the normal component of $\boldsymbol{B}$ is the same on both sides of the sheet. According to the circuital law, however, the tangential component is not conserved and a line of $\boldsymbol{B}$ is deflected in the direction shown.

= 0 by considering path a in Figure
this path is zero, and the line integral of
also zero. Now, since the line integrals
($B_\rho = 0$), the line integrals along sides
les 3 and 4 can each be situated at any
and B_z must therefore be either zero or

e neither beginning nor end, and since
finite space outside must equal the finite
must be zero outside.
ked once by the current, and, outside

$$B_\varphi = \frac{\mu_0 I}{2\pi\rho}. \tag{7-88}$$

he line integral of $B_\varphi\, dl$ over a circle
of the small cylinder shown in Figure
t be zero according to Eq. 7-75 because
by the path.
in Figure 7-16, and remembering that
side, and that $B_z = 0$ outside, we see

$$B_z = \mu_0 N'I. \tag{7-89}$$

inside a long solenoid in the region
efore uniform and equal to μ_0 times the
ter.

LINES OF $\boldsymbol{B}$ AT A

sheet carrying a current density of λ
7-17. We can find how the lines of $\boldsymbol{B}$

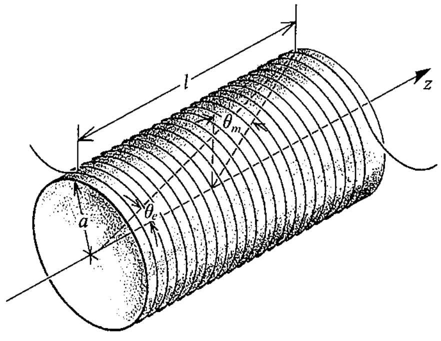

Figure 7-18. A short solenoid.

are refracted in passing through the sheet by proceeding as in Section 4.1.2. Since the divergence of $\boldsymbol{B}$ is zero, the normal component of $\boldsymbol{B}$ is conserved:

$$B_{n1} = B_{n2}. \tag{7-90}$$

Also, if we apply Ampère's circuital law to a path of length L that is perpendicular to the sheet as in Figure 4-2,

$$B_{t_1}L - B_{t_2}L = \mu_0 \lambda L, \tag{7-91}$$

$$B_{t_2} = B_{t_1} - \mu_0 \lambda. \tag{7-92}$$

The line of force is therefore rotated in the clockwise direction for an observer looking in the direction of the vector $\boldsymbol{\lambda}$.

We could have arrived at this result in another way. The magnetic induction $\boldsymbol{B}$ is due to the current sheet itself and the other currents flowing elsewhere in the system. According to Ampère's circuital law, the current sheet produces just below itself in Figure 7-17a $\boldsymbol{B}$ that is directed to the left and whose magnitude is $\mu_0\lambda/2$. Similarly, the $\boldsymbol{B}$ just above the sheet is directed to the right and has the same magnitude. If we add this field to that of the other currents, we see that the tangential components of $\boldsymbol{B}$ must differ as above.

Example

THE SHORT SOLENOID

We can calculate B on the axis of a short solenoid by summing the contributions of the individual turns, using Eq. 7-19. If the length of the solenoid is l and if its radius is a, the magnetic induction at the center is

$$B = \frac{\mu_0}{2} \int_{-l/2}^{+l/2} \frac{a^2 N' I\, dz}{(a^2 + z^2)^{3/2}}, \tag{7-93}$$

$$= \mu_0 N' I \sin \theta_m, \tag{7-94}$$

as in Figure 7-18. We have assumed that the solenoid is close wound. For a long solenoid $\theta_m \rightarrow \pi/2$, and $B \rightarrow \mu_0 N'I$, as in the example on page 315.

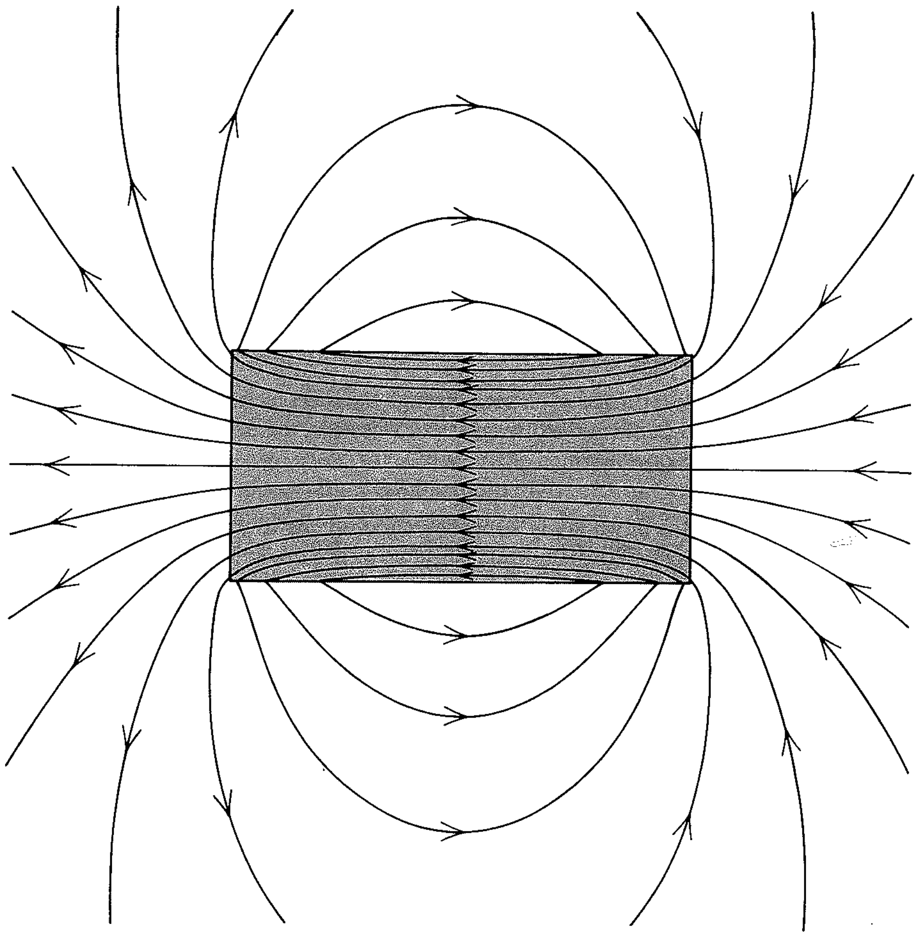

Figure 7-19. Lines of $\boldsymbol{B}$ for a solenoid whose length is equal to twice its diameter.

At one end, again on the axis,

$$B = \mu_0 N'I \frac{\sin \theta_e}{2}. \tag{7-95}$$

The magnetic induction thus decreases at both ends of the solenoid, and this is of course due to the fact that the lines of $\boldsymbol{B}$ flare out as in Figure 7-19.

Upon crossing the current sheet, the radial component of $\boldsymbol{B}$ remains unchanged, but the axial component changes both its magnitude and its sign. For example, the axial component in the upper left-hand side of the solenoid in Figure 7-19 changes from, say, $-0.9\ \mu_0 N'I$ to $+0.1\ \mu_0 N'I$, since the surface current density λ is $N'I$.