Kalman Filters Summary

Kalman described a filter in 1960 that utilizes the past best guess, the dynamics(described by the state equation), the noisy measurement of the output, covariance matrices of corrupting noise added into the states, and noise on the measurement of the outputs to come up with an optimal guess on what the next states are. Our derivations assumed the random processes, were zero mean, white Guassian noise, and that the noise on the states, was independent of the noise on the measurement. It is uses a predictor, corrector method, where the optimum balance between the predicted new state and the probably conflicting noisy measurement is obtained, and refined as more measurements are made. This is intended as a summary to supplement your in class derivation, and to try to bring everyone to the same place of understanding.

You derived the method for a first order state equation, like an RC circuit. The results are given below for you to check with. It is assumed you have the Kalman Filters handout outlining the derivation handy:

- $\tilde{x}$ is the predicted state. $\hat{x}$ is the corrected state. w is the noise on the state. v is the noise on the measurement. $Q \equiv E[w^2]$ and $R \equiv E[v^2]$.
- The result of step three for the Kalman gain, K is:

$$K_k = \frac{CE[(x_k - \tilde{x}_k)^2]}{C^2 E[(x_k - \tilde{x}_k)^2] + R}$$

- It is common to define

$$P_{k|k} \equiv \hat{P}_k \equiv E[(x_k - \hat{x}_k)^2]$$

  and

$$P_{k|k-1} \equiv \tilde{P}_k \equiv E[(x_k - \tilde{x}_k)^2]$$

  where the subscript $k|k-1$ or tilde is for the predictor, and $k|k$ or hat is for the corrected state (using the measured output, and optimized Kalman gain).

- This makes

$$K_k = \frac{CP_{k|k-1}}{C^2 P_{k|k-1} + R}$$

- From the result of four,

$$P_{k|k-1} = A^2 P_{k-1|k-1} + Q$$

and from step 2 of the derivation handout:

$$P_{k|k} = (1 - K_k C)^2 P_{k|k-1} + K_k^2 R$$

The procedure to do the Kalman filter reduces to:

- Get your best guess at an initial position and mean square error between this position, and what the real position is. These are $\hat{x}_0$ and $P_{0|0}$.
- Get the predictor for the state and the mean square error (Priori):

  $$\tilde{x}_k = A\hat{x}_{k-1} + Bu_{k-1}$$

  and

  $$P_{k|k-1} = A^2 P_{k-1|k-1} + Q$$

- Do the corrector updates (Posteriori):

  $$K_k = \frac{CP_{k|k-1}}{C^2 P_{k|k-1} + R},$$

  $$P_{k|k} = (1 - K_k C)^2 P_{k|k-1} + K_k^2 R,$$

  and

  $$\hat{x}_k = \tilde{x}_k + K_k(y - C\tilde{x}_k)$$

- Then you repeat the predictor and corrector steps with incremented k.