# Information Basics

The purpose of this exercise is to begin to understand information better. We will be investigating how we might quantify information. For example, maybe after we figure this out, we can figure out how much information you are learning in each of your classes. In your group see if you can answer the following questions:

1. How many distinct numbers can you represent with one base ten digit? How about two? How about three? What mathematical expression would give you the number of numbers for an $n$ digit base ten number?
2. If the maximum number that can be represented by a base $b$ number is $M$, what is the mathematical expression for the minimum number of digits, $n$, needed to express that number in base $b$?
3. On the average, with the same size font and pages, would an English book or a Russian book have more information per page? Hint: Russian has 33 characters in their alphabet, and English has 26. Is the amount of information per per character the same in each? Is information per character a good unit when comparing these two? Why or why not? We are looking for some kind of measure. What do you propose we do?
4. If you trying to guess a random integer between zero and seven, what is the minimum number of yes/no questions you must ask to find it out for certain? What is an example set of those questions?
    1. What if the probabilities for each number were not equal, but instead given by the following table?

| Unknown Number | Probability of that Number |
|---|---|
| 0 | 1/2 |
| 1 | 1/4 |
| 2 | 1/8 |
| 3 | 1/16 |
| 4 | 1/32 |
| 5 | 1/64 |
| 6 | 1/128 |
| 7 | 1/128 |

    2. What would your questions be now?
    3. On the average, would you find the number faster with this probability distribution, or when they are all equally likely?
    4. What does that say about how much information you get on the average per question?
5. Suppose we have a regular die (six sides, with numbers, one through six printed on each of its six sides respectively).
    1. How can we measure the amount of information we get about what number turned up when we roll it? Give a mathematical expression for the amount of information received from learning the result that a particular number ending up on top.
    2. What would the average amount of that information be? Give the mathematical expression for this too.
6. Suppose you have an experiment where there are N possible outcomes. If that is all you know, and you want to find the probabilities that are most honest in the sense that running the experiment gives you the maximum information in an average sense from the result of the experiment, what would the probabilities be? Hint: Your intuition is probably good here, but you can prove this using Lagrange multipliers using the constraint that all the probabilities must add up to one. Give it a try!