TRANSMISSION LINE ANALYSIS

The object of this book is to build a model for the system represented in the figure① and to use the model to predict useful characteristics and modes of operation of the system. The reader is assumed to have a good understanding of phasors and eigenvalue techniques.

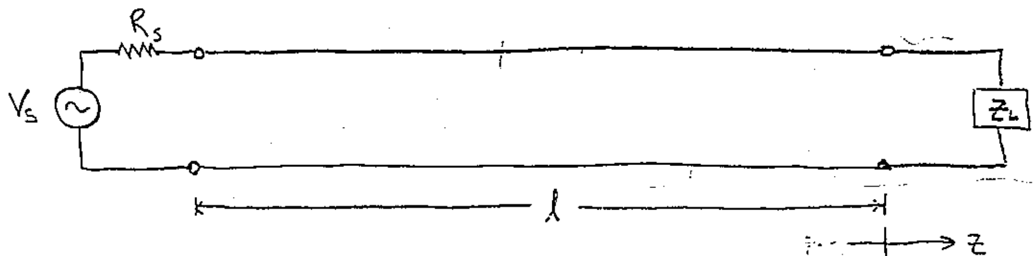


Figure ①

We will attemt to find the Voltage $V(z)$ and the current $I(z)$ for $0<z<l$. To start we need a model for the transmission lines that takes into account the capacitive, inductive, resistive, and conductive properties of the cable. The one depicted in figure ② will yield good results.

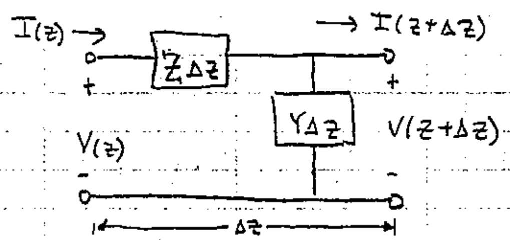


figure ②

Note that $I(z)$ and $V(z)$ are phasors thus $V(z,t) = Re[V(z)e^{j\omega t}]$ and $I(z,t) = Re[I(z)e^{j\omega t}]$. Also note that figure ② represents a section of the transmission lines Δz long. Y models the conductive capacitive characteristics of the line and is defined as follows:

$$Y = g + j\omega c \text{ mho's}$$

where g and c are per unit length values so that when multiplied by Δz give the conductance and capacitance for Δz of transmission line. Z, which should not be confused with the z-axis as defined in figure ①, models the resistance and inductance of the transmission line. It is defined as follows:

$$Z = r + j\omega L \text{ ohms}$$

L and r are also defined as per unit length for the same reason as g and C. Using this model solutions are derived as follows.

Using KCL and KVL from circuit theory the following equations are obtained from figure ②

$$E① \quad I(z+\Delta z) - I(z) = -Y(\Delta z)\cdot V(z-\Delta z) \quad KCL$$

$$E② \quad V(z+\Delta z) - V(z) = -Z(\Delta z)\cdot I(z) \quad KLV$$

E① and E② are divided by Δz and the limit as $\Delta z \to 0$ is taken recalling the definition of the derivitive and noting that the $\lim_{\Delta z \to 0} f(z-\Delta z) = f(x)$ for any linear function f(x) (as well as many nonlinear functions). In this way E③ and E④ are derived.

$$\lim_{\Delta z \to 0}\left(\frac{I(z+\Delta z)-I(z)}{\Delta z}\right) = \lim_{\Delta z \to 0}\left(-Y\,V(z+\Delta z)\right)$$

$$E③ \quad \frac{dI(z)}{dz} = -YV(z)$$

$$\lim_{\Delta z \to 0}\left(\frac{V(z-\Delta z)-V(z)}{\Delta z}\right) = -ZI(z)$$

$$E④ \quad \frac{dV(z)}{dz} = -ZI(z)$$

These equations are arranged in a matrix as follows:

$$E⑤ \quad \frac{d}{dz}\begin{bmatrix} I \\ V \end{bmatrix} = \begin{bmatrix} 0 & -Y \\ -Z & 0 \end{bmatrix}\begin{bmatrix} I \\ V \end{bmatrix}$$

note- This is a 2nd order system of differential

At this point a solution is assumed to be $Ce^{-\gamma z}$ to these equations. By rearranging E⑤ this solution will be verified using an eigen value approach. To rearrange E⑤ it is noted that for the assumed solution of $Ce^{-\gamma z}$ the derivative with respect to z (i.e. d/dz) is $-\gamma$. In other words, to differentiate the assumed solution just multiply by $-\gamma$.

Replacing $d/dz \to -\gamma$ in E⑤ gives:

$$-\gamma \begin{bmatrix} I \\ V \end{bmatrix} = \overbrace{\begin{bmatrix} 0 & -Y \\ -Z & 0 \end{bmatrix}}^{A} \begin{bmatrix} I \\ V \end{bmatrix}$$

*note- The matrix is defined as the A matrix.

dividing by -1 and bring everything to the same side and then factoring out the [I V]' vector gives:

E⑥ $$\left(\gamma I - \begin{bmatrix} 0 & +Y \\ +Z & 0 \end{bmatrix} \right) \begin{bmatrix} I \\ V \end{bmatrix} = 0$$

*note - I is the matix 1 and is multiplied by γ so the A matrix can be subtracted from it.

For this equation to have none trival solutions $|\gamma I - A| = 0$. This is calculated as follows:

$$\left| \begin{bmatrix} \gamma & 0 \\ 0 & \gamma \end{bmatrix} - \begin{bmatrix} 0 & +Y \\ +Z & 0 \end{bmatrix} \right| = 0$$

$$\Rightarrow \begin{vmatrix} \gamma & -Y \\ -Z & \gamma \end{vmatrix} = 0$$

$\Rightarrow \quad \gamma^2 - ZY = 0$ or $\gamma = \pm\sqrt{ZY}$ and is defined as follows: E⑦ $\gamma \equiv {}^{+}\sqrt{ZY}$

This means that the assumed solution of $Ce^{-\gamma z}$ is valid when $\gamma = \sqrt{ZY}$. Recalling that phasors where used to redefine the current $I(z,t)$ and Voltage $V(z,t)$ to $Re[I(z)e^{j\omega t}]$ and $Re[V(z)e^{j\omega t}]$ and then solutions to $V(z)$ and $I(z)$ were derived, $I(z,t)$ and $V(z,t)$ can be expressed as follows:

$$E⑧ \quad V(z,t) = Re\left[C_1 e^{-\gamma z} e^{j\omega t}\right] = Re\left[C_1 e^{j\omega t - \gamma z}\right]$$

$$E⑨ \quad I(z,t) = Re\left[C_2 e^{-\gamma z} e^{j\omega t}\right] = Re\left[C_2 e^{j\omega t - \gamma z}\right]$$

These equation can be further modified to make them more understandable and useful if it is recalled that for a second order ODE, there are two independent solutions. These appeared in the previous analysis as the two values which would satisfy the assumed solution which were $\pm\sqrt{ZY}$. Now the value of C will come from the forcing function and will be defined as $V_+, V_-, I_+,$ and I_-. We will asign V_+ and I_+ to $-\sqrt{ZY}$ and will physically interpret this as the forward going wave after we have written out the equations as follows:

$$E⑩ \quad V(z,t) = Re\left[V_+ e^{-\sqrt{ZY}z} e^{j\omega t} + V_- e^{+\sqrt{ZY}z} e^{j\omega t}\right]$$

$$E⑪ \quad I(z,t) = Re\left[I_+ e^{-\sqrt{ZY}z} e^{j\omega t} + I_- e^{+\sqrt{ZY}z} e^{j\omega t}\right]$$

Since these equations are coupled, in other words $I(z,t)$ and $V(z,t)$ are related and cannot be chosen independently, it is necessary and will prove to be useful to find this relation. Solving for the eigen vectors will give this relation. Plugging in the values obtained for γ which is reffered to as the propagation Constant into E⑥ the dependence between V and I is determined. First, to find the relation between I_+ and V_+ use $+\gamma$ as specified in E⑩ and E⑪ and realizing that d/dz of of $V_+(z,t)$ or $I_+(z,t) = -\gamma V(z,t)$ or $-\gamma I(z,t)$ (respectively) is used to arrange E⑤ into the following:

$$\begin{bmatrix} -\sqrt{ZY} & Y \\ -Z & \sqrt{ZY} \end{bmatrix} \begin{bmatrix} I \\ V \end{bmatrix} = 0 \Rightarrow \begin{array}{l} E⑫ \quad \sqrt{ZY}\, I - YV = 0 \\ E⑬ \quad -ZI + \sqrt{ZY}\, V = 0 \end{array}$$

solving either E⑫ or E⑬ gives:

$$I = \frac{Y}{\sqrt{ZY}} V \Rightarrow I = \sqrt{\frac{Y}{Z}}\, V \quad \text{or rearranged} \quad V = \sqrt{\frac{Z}{Y}}\, I$$

The expression $\sqrt{\frac{Z}{Y}}$ derived as the relation between I and V appears so often and is so useful it is given its own symbol and name.

D① $\quad Z_0 = \frac{1}{Y_0} = \sqrt{\frac{Z}{Y}} \qquad Z_0$ = characteristic impedance

This is the dependence of the particular solution $-\gamma$ which corresponds to V_+ and I_+. Finding the eigen vector of $+\gamma$ would give the relation between V_- and I_-. It is given without derivation along with the relation between I_+ and V_+.

E⑭ $\quad V_+ = Z_0 I_+ \qquad$ from above

E⑮ $\quad V_- = -Z_0 I_+ \qquad$ stated without derivation

It is convienient to rearranged E⑭ as E⑯:

E⑯ $$\begin{bmatrix} V_+ \\ I_+ \end{bmatrix} = \begin{bmatrix} V_+ \\ V_+/Z_0 \end{bmatrix} = V_+ \begin{bmatrix} 1 \\ Z_0 \end{bmatrix}$$

E⑮ is similarly rearranged into E⑰:

E⑰ $$\begin{bmatrix} V_- \\ I_- \end{bmatrix} = \begin{bmatrix} V_- \\ -V_-/Z_0 \end{bmatrix} = V_- \begin{bmatrix} 1 \\ \frac{-1}{Z_0} \end{bmatrix}$$

With these rearrangements all the information so far accumulated can be summarized as follows recalling that $V_{(z)}$ and $I_{(z)}$ are the superposition of V_+, I_+ and V_-, I_- (i.e. E⑯ and E⑰).

E⑱ $$\begin{bmatrix} V(z) \\ I(z) \end{bmatrix} = V_+ \begin{bmatrix} 1 \\ \frac{1}{Z_0} \end{bmatrix} e^{-\gamma z} + V_- \begin{bmatrix} 1 \\ \frac{-1}{Z_0} \end{bmatrix} e^{\gamma z}$$

Now it is convienient to define a scattering matrix which will be, in this case, a 1x1 matrix with its only term being a reflection term. It is defined as follows:

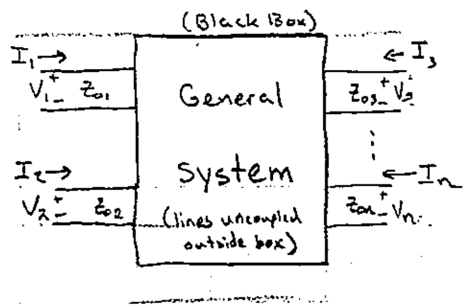
D② $\quad \underline{b} = \underset{\sim}{S}\underline{a} \qquad$ where $\underset{\sim}{S}$ is scattering matrix

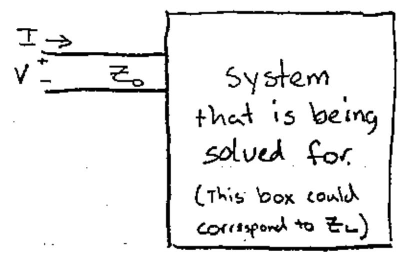
The ~ under the 'S' denotes that 'S' is a matrix. As previously noted $\underset{\sim}{S}$ will will turn out to be a 1x1 for this case but the derivation for the general nth order case is solved it what follows and then the parameters will be plugged in and it will reduce to the 1x1. $\underline{a}$ and $\underline{b}$ are defined as follows where the – under them denotes that they are vectors.

D③ $\underline{b} = \underset{\sim}{Z_o}^{-1/2}\, \underline{V_-}$ ← bar is a reminder that in the general case V_- is a vector

D④ $\underline{a} = \underset{\sim}{Z_o}^{-1/2}\, \underline{V_+}$

Reffer to the following block diagram for definitions of parameters:





IN General:

$$V_1 = V_{1+} + V_{1-}$$
$$V_2 = V_{2+} + V_{2-}$$
$$V_3 = V_{3+} + V_{3-}$$
$$V_n = V_{n+} + V_{n-}$$
$$\rightarrow \quad \underline{V} = \underline{V_+} + \underline{V_-}$$

where:

$$\underline{V_+} = \begin{bmatrix} V_{1+} \\ V_{2+} \\ \vdots \\ V_{n+} \end{bmatrix} \qquad \underline{V_-} = \begin{bmatrix} V_{1-} \\ V_{2-} \\ \vdots \\ V_{n-} \end{bmatrix}$$

In this case:

$\underline{V} = \underline{V_+} + \underline{V_-}$ where all vectors are 1x1 or: $V = V_+ + V_-$

In like manner $\underline{I} = \underline{I_+} + \underline{I_-}$

In this case

now
$$\underset{\sim}{Z_o} = \begin{bmatrix} Z_{o1} & & 0 \\ & Z_{o2} & \\ 0 & & \ddots \; Z_{on} \end{bmatrix}$$

$\underset{\sim}{Z_o} = [Z_o]$ a 1x1 matrix

by Taylor expansion:

$$\underset{\sim}{Z_o}^{-1/2} = \begin{bmatrix} Z_{o1}^{-1/2} & & 0 \\ & Z_{o2}^{-1/2} & \\ 0 & & \ddots \; Z_{on}^{-1/2} \end{bmatrix}$$

In this case

$$\underset{\sim}{Z_o}^{-1/2} = [Z_o^{-1/2}] = Z_o^{-1/2}$$

This completes this set of definitions. They may seem a bit awkward but they will be very handy when computing power and the work fine for the purposes of this book so the investment in learning them is a good one. With one last definition the scattering matrix will be solved for.

D⑤ $\underset{\sim}{Z}_L$ is defined by $\underline{V} = \underset{\sim}{Z}_L \underline{I}$

This is simply a matrix/vector statement of Ohms Law relating the ith input line into the black box by an impedance. Thus for our case were the black box is Z_L D⑤ reduces to $V = Z_L I$.

Solving for the scatting matrix is done as follows:

$$\underline{V} = \underset{\sim}{Z}_L \underline{I} = \underline{V}_+ + \underline{V}_-$$

recall $\underline{a} \equiv \underset{\sim}{Z}_0^{-1/2} \underline{V}_+ \Rightarrow \underline{V}_+ = \underset{\sim}{Z}_0^{1/2} a$

$$\underline{b} \equiv \underset{\sim}{Z}_0^{-1/2} \underline{V}_- \Rightarrow \underline{V}_- = \underset{\sim}{Z}_0^{1/2} b$$

then $\underline{V} = \underset{\sim}{Z}_0^{1/2} \underline{a} + Z_0^{1/2} \underline{b}$ but $\underline{b} = \underset{\sim}{S} \underline{a}$

so $\underline{V} = \underset{\sim}{Z}^{1/2} \underline{a} + \underset{\sim}{Z}_0^{1/2} \underset{\sim}{S} \underline{a} = \underset{\sim}{Z}_0^{1/2} (\underset{\sim}{I} + \underline{S}) \underline{a}$

A similar derivation for $\underline{I}$ yields:

$$\underline{I} = \underset{\sim}{Z}_0^{-1} V_+ - \underset{\sim}{Z}_0^{-1} V_- = \underset{\sim}{Z}_0^{-1} \left[\underset{\sim}{Z}^{1/2} \underline{a} - Z_0^{1/2} \underline{b} \right] = \underset{\sim}{Z}_0^{-1/2} \left[\underset{\sim}{I} - \underset{\sim}{S} \right] \underline{a}$$

This is the critical step for this boundry condition:

E⑲ $\underline{V} = \underset{\sim}{Z}_L \underline{I} \Rightarrow Z_0^{1/2} (\underset{\sim}{I} + \underset{\sim}{S}) \underline{a} = \underset{\sim}{Z}_L \underset{\sim}{Z}_0^{-1/2} (\underset{\sim}{I} - \underset{\sim}{S}) \underline{a}$

solving this for $\underset{\sim}{S}$ by equating coeficients gives

$$\left(\underset{\sim}{Z}_0^{1/2} + \underset{\sim}{Z}_L * \underset{\sim}{Z}_0^{-1/2} \right) \underset{\sim}{S} = \underset{\sim}{Z}_L * \underset{\sim}{Z}_0^{-1/2} - \underset{\sim}{Z}_0^{1/2}$$

E⑳ $\underset{\sim}{S} = \left(\underset{\sim}{Z}_0^{1/2} + \underset{\sim}{Z}_L * \underset{\sim}{Z}_0^{-1/2} \right)^{-1} \left(\underset{\sim}{Z}_L * \underset{\sim}{Z}_0^{-1/2} - Z_0^{1/2} \right)$

This is a very useful result. We only need to construct $\underset{\sim}{Z}_0^{1/2}$, $\underset{\sim}{Z}_L$, and $\underset{\sim}{Z}_0^{-1/2}$ and we can then calculated the scattering matrix. These are easy to construct for a wide range of problems and very easy for the problem being solved in this book.

$\underset{\sim}{Z}_o^{-1/2}$, $\underset{\sim}{Z}_o^{1/2}$, $\underset{\sim}{Z}_L$ are calculated as follows:

$$\underset{\sim}{Z}_o = \begin{bmatrix} Z_{o1} & & 0 \\ & \ddots & \\ 0 & & Z_{on} \end{bmatrix}$$ where in our case $n=1$ so

$$\underset{\sim}{Z}_o = [Z_o] = Z_o$$ a 1X1 matrix and a number have same characteristics

then $\underset{\sim}{Z}_o^{1/2} = [Z_o^{1/2}] = \sqrt{Z_o}$ and $\underset{\sim}{Z}_o^{-1/2} = [Z_o^{-1/2}] = \frac{1}{\sqrt{Z_o}}$

Now $\underset{\sim}{Z}_L = \underline{V}\,\underline{I}^{-1}$ or $Z_{L_i} = \frac{V_{L_i}}{I_{L_i}}$ thus $\underset{\sim}{Z}_L = \begin{bmatrix} Z_{L_1} & & & 0 \\ & L_{L_2} & & \\ & & \ddots & \\ 0 & & & L_{Ln} \end{bmatrix}$

where, again, $n=1$ thus $\underset{\sim}{Z}_L = [Z_L] = Z_L$ for our case

Having defined these $\underset{\sim}{S}$ is:

$$\underset{\sim}{S} = (\sqrt{Z_o} + Z_L/\sqrt{Z_o})^{-1}(Z_L\sqrt{Z_o} - \sqrt{Z_o})$$

rearranging gives:

$$\text{E (21)} \qquad \underset{\sim}{S} = \frac{Z_L - Z_o}{Z_L + Z_o}$$

This is a very important result. It can be seen the $\underset{\sim}{S}$ has only one term in this case. It is a reflection term which give the ratio of reflected to incident wave at the load. This can be derived from classical methods and is called Γ_L. Classically it is defined as follows:

$$\text{D (6)} \qquad \Gamma_L \equiv \frac{V_{-L}}{V_{+L}}$$ and is equal to $\underset{\sim}{S}$ in this case since there is no transmission.

To show this is the case the derivation follows by classical means. Recalling and rewriting E (19) with the preceding definition gives:

$$\text{E (22)} \qquad \begin{bmatrix} V(0) \\ I(0) \end{bmatrix} = V_+ \left[\begin{bmatrix} 1 \\ \frac{1}{Z_o} \end{bmatrix} \cancelto{1}{e^{-\gamma(0)}} + \Gamma_L \begin{bmatrix} 1 \\ \frac{-1}{Z_o} \end{bmatrix} \cancelto{1}{e^{\gamma(0)}} \right]$$

Z is set to zero since that is where the load is.

At this point it is a simple matter to write out the equation, taking them out of vector/matrix form and divide the expression for V_L by I_L and set this equal to Z_L (Ohm's law) and solve for Γ_L. This is done as follows:

$$V_{(0)} = V_+\left[1 + \Gamma_L\right]$$

$$I_{(0)} = V_+\left[1/Z_0 - \frac{\Gamma_L}{Z_0}\right]$$

now $\frac{V(0)}{I_{(0)}} = Z_L \Rightarrow \frac{V_+\left[1+\Gamma_L\right]}{V_+\left[\frac{1}{Z_0} - \Gamma_L/Z_0\right]} = Z_L$

E(23) $\Rightarrow \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$ which = $\underset{\sim}{S}$ derived earlier E(21)

It must be noted the Γ_L is a reflection coefficient while $\underset{\sim}{S}$ is a matrix that can have terms that are reflection coefficients as well as terms that are transmission coefficients.

We will use this classical approach to find $\Gamma(z)$. By definition it is as follows:

E(24) $$\Gamma(z) = \frac{V_-(z)}{V_+(z)} = \frac{-I_-(z)}{I_+(z)}$$

from E(18) we know that $V_-(z) = V_- e^{\gamma z}$
and $V_+(z) = V_+ e^{-\gamma z}$

$$\Gamma(z) = \frac{V_- e^{\gamma z}}{V_+ e^{-\gamma z}} = \frac{V_-}{V_+} e^{2\gamma z}$$

we know that $\frac{V_-}{V_+} = \Gamma_L$ thus

E(25) $$\Gamma(z) = \Gamma_L e^{2\gamma z}$$

We can use this to redefine E(18) as follows:

E(26) $$\begin{bmatrix} V(z) \\ I(z) \end{bmatrix} = V_+ e^{-\gamma z}\left[\begin{bmatrix} 1 \\ \frac{1}{Z_0} \end{bmatrix} + \Gamma(z)\begin{bmatrix} 1 \\ \frac{-1}{Z_0} \end{bmatrix}\right]$$

At this point we will take the equations out of the matrix/vector notation and use them to solve for $Z(z)$. where $Z(z)$ is as follows:

D ⑦ $$Z(z) = \frac{V(z)}{I(z)}$$

Forming two equations from E ㉖ gives:

E ㉗ $$V(z) = V_+ e^{-\gamma z}\left[1 + \Gamma(z)\right]$$

E ㉘ $$I(z) = V_+ e^{-\gamma z}\left[\frac{1}{Z_0} - \Gamma(z)/Z_0\right]$$

Next these equations are divided to find $Z(z)$ as specified in D⑦.

$$Z(z) = \frac{V(z)}{I(z)} = \frac{V_+ e^{-\gamma z}\left[1 + \Gamma(z)\right]}{V_+ e^{-\gamma z}\left[\frac{1}{Z_0} - \Gamma(z)/Z_0\right]}$$

$$= \frac{\left[1 + \Gamma(z)\right]}{\left[\frac{1}{Z_0} - \Gamma(z)/Z_0\right]}$$

multiplying by Z_0/Z_0 gives

$$Z(z) = Z_0\,\frac{1+\Gamma(z)}{1-\Gamma(z)}$$

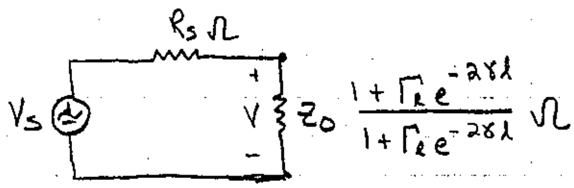
or

E ㉙ $$Z(z) = Z_0\,\frac{1 + |\Gamma_l| e^{2\gamma z}}{1 - |\Gamma_l| e^{2\gamma z}}$$

To consider this problem fully solved it is neccessary to solve for V_+ in terms of V_s, R_s, Z_L, l, L, G, C, R. To do this we find the equivalent impedance at the source end using the equation for impedance as a function of z and plugging in $-l$ to find it at the source end.

recall: $$Z(z) = Z_0 \frac{1 + \Gamma(z)}{1 - \Gamma(z)} = Z_0 \frac{1 + \Gamma_L e^{+2\gamma z}}{1 - \Gamma_L e^{+2\gamma z}}$$

A Thevinin equivilent circuit is constructed as follows:



Soluing for V by voltage divider techniches:

E 30 $$V = \frac{Z_{eq}}{Z_{eq} + R_s} V_s$$

where $$Z_{eq} = Z_0 \frac{1 + \Gamma_L e^{-2\gamma l}}{1 + \Gamma_L e^{-2\gamma l}}$$

However $V = V_+(-l)$ so we can solve for V_+ as

$$V_+ e^{-\gamma(-l)} = \frac{Z_{eq}}{Z_{eq} + R_s} V_s$$

E 31 $$V_+ e^{\gamma l} = \frac{Z_{eq}}{Z_{eq} + R_s} V_s \Rightarrow V_+ = \frac{Z_{eq}}{Z_{eq} + R_s} e^{-\gamma l} V_s$$

We find V_+ as a function of time at $-l$ by rearranging and taking the inverse phasor transform:

E 32 $$V_+(t)\Big|_{-l} = Re\left[V_+ e^{j\omega t}\right]\Big|_{-l} = Re\left[\frac{Z_{eq}}{Z_{eq} + R_s} e^{-\gamma l} V_s e^{j\omega t}\right]$$

The Smith Chart

Given the need or desire to find V, I and Z at different points along the transmission line and the form of the equations for finding the values it is no wonder that with no computationally intensive devices like the computer that a graphical method was derived to get the desired values (analog computer?). The basics of the smith chart can be understood as follows:

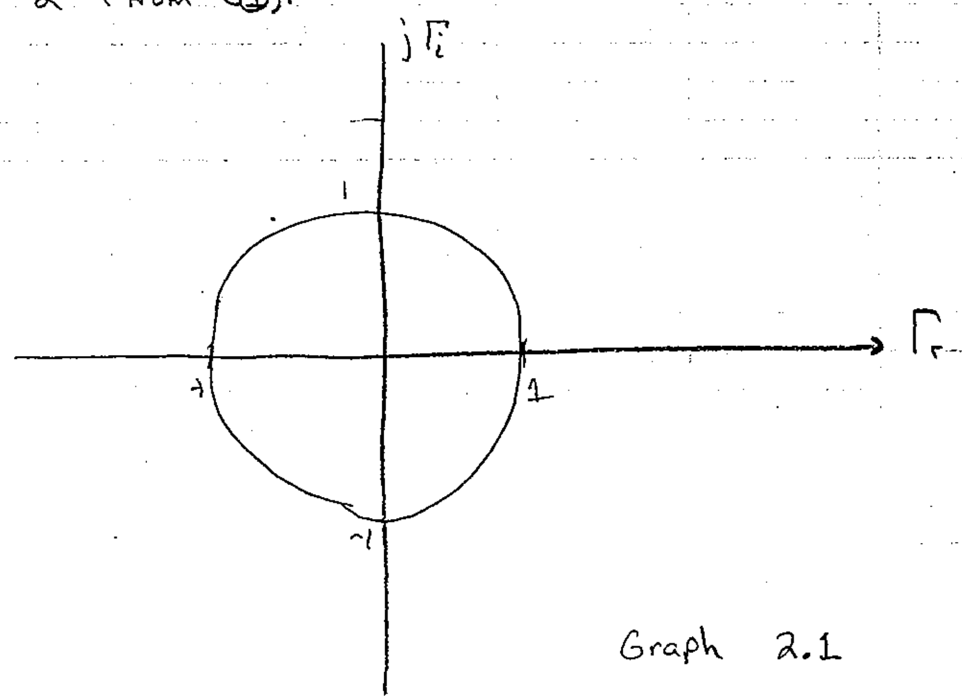
$$Z(z) = Z_0 \frac{1 + \Gamma(z)}{1 - \Gamma(z)} \qquad \text{chapter 1} \quad E\ (29)$$

$$\text{where } \Gamma(z) = |\Gamma_L| e^{2\gamma z} \qquad \text{where } |\Gamma_L| = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\text{now} \qquad |\Gamma_\ell| = \left| \frac{Z_\ell - Z_0}{Z_L + Z_0} \right| \leq 1$$

C(1) so $|\Gamma(z)| < 1$ which is the basis for the Smith chart

we see that $\Gamma(z) = \Gamma_r + j\Gamma_i$ so we make a complex graph with Γ_r and Γ_i our coordinates realizing that the graph will always be in a circle of diameter 2 (from C(1)).



Graph 2.1

Next we see that we can rearrange E 29 as follows:

$$E\,33 \qquad \frac{Z}{Z_0} = \frac{1+\Gamma(z)}{1-\Gamma(z)} \equiv z$$ which we will call the normalized impedance

we will plot $\Gamma(z)$ on graph 2.1

$$E\,34 \qquad z = \frac{Z}{Z_0} = \frac{R_{(z)} + j\,X_{(z)}}{Z_0} = r + jx$$

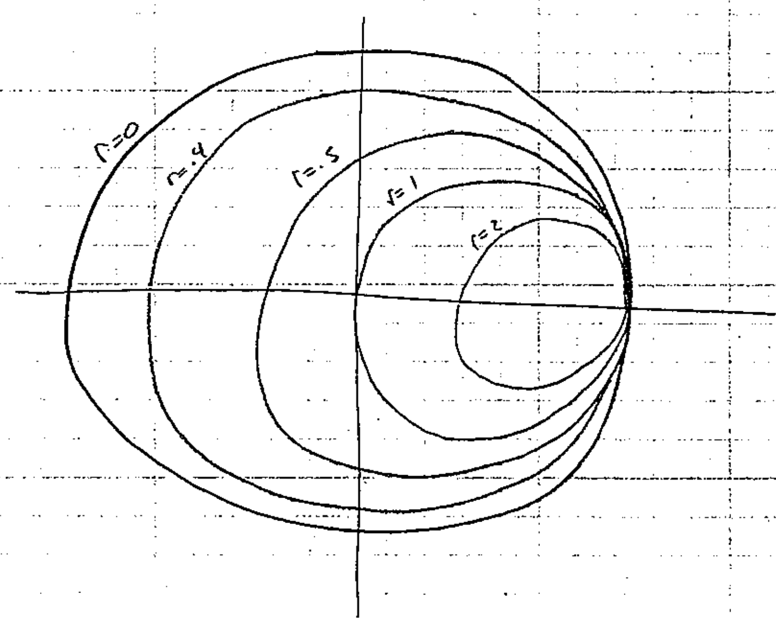
$$E\,35 \qquad \Gamma(z) = \frac{z-1}{z+1} = \Gamma_{(z)\,r} + j\,\Gamma_{(z)\,i}$$

substituting E 35 → E 34

$$E\,36 \qquad r = \frac{1-\Gamma_{(z)r} - \Gamma_{(z)i}^{\,2}}{(1-\Gamma_{(z)r})^2 + \Gamma_{(z)i}} \Rightarrow \left(\Gamma_{(z)r} - \frac{r}{1+r}\right)^2 + \Gamma_{(z)i}^{\,2} = \left(\frac{1}{1+r}\right)^2$$

$$E\,37 \quad \text{and} \quad x = \frac{2\,\Gamma_{(z)i}}{(1-\Gamma_{(z)r})^2 + \Gamma_{(z)i}^{\,2}} \Rightarrow \left(\Gamma_r - 1\right)^2 + \left(\Gamma_{(z)i} - \frac{1}{x}\right)^2 = \left(\frac{1}{x}\right)^2$$

E 36 gives a family of curves as follows



And E 37 plots a family of orthoganal curves to E 36.

The normalized impedance can be calculated from any value of $\Gamma(z)$ on the graph by the formula:

$$z = \frac{Z_L}{Z_0} = \frac{1+\Gamma(z)}{1-\Gamma(z)}$$

Also, recall that ρ the standing wave ratio is defined as follows

$$\rho = \frac{|V_{max}|}{|V_{min}|} = \frac{|I_{max}|}{|I_{min}|} = \frac{1+|\Gamma(z)|}{1-|\Gamma(z)|}$$

now $|\Gamma(z)| = \frac{\rho-1}{\rho+1}$ thus SWR are circles on the $\Gamma(z)$ graph.

To find V_{max} and V_{min} note that:

$V(max)$ occurs at $z_{max} = \rho$,
V_{min} occurs at $z_{min} = 1/\rho$

Thus $V(max)$ occurs everytime you are at a point in the line corresponding to .125 λ or the right side of the graph. To find how far this is from the load subtract the point (in wavelengths) of the normalized impedance. Same for V_{min} only it occurs at .5 λ.

Return loss can be calculated by drawing a line down from where the ρ circle meets the real axis and reading where it intersects the RLS. Same with transmitted Power, Read of the TPS. These both are in dB.

We also want to match impedance with this. we want $\Gamma(z)=1$ so we plot impedance or admittance and we find the distance in wavelengths to the $\rho=1$ circle. That gives us on parameter. Then we find the distance we need to go in the ij direction and we then measure that in wavelengths for our second parameter.

Who read this?